Distributed aperture effect in laser rods with negative lenses: a discussion

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The difference between a ray transfer matrix in a lenslike medium and a section of lenslike medium immersed in vacuum is demonstrated. The distributed aperture and useful volume are calculated for a laser rod with negative lenses ground on the ends, using a multiplication of ray transfer matrices and the deductive method used by Barnes and Scalise. The results we obtained are different from their results.

Introduction

First, the *ABCD* matrices of the lenslike medium and of a section of lenslike medium immersed in vacuum are deduced by means of the differential equation representing the transmission of light through an inhomogeneous medium. Then the distributed aperture and useful volume are obtained by multiplication of the matrices.

Ray Transfer Matrix for a Lenslike Medium

The refractive-index distribution in a laser rod pumped homogeneously is the same as that in a lenslike medium, namely,

$$n = n_0 - n_2(x^2/2), \tag{1}$$

where x is the perpendicular distance of the ray from the optical axis, n_0 is the constant coefficient of the refractive index, and n_2 is the parabolic coefficient of the refractive index $n_2 > O$.

The differential equation of light rays in an inhomogeneous medium[1] is

$$\frac{d}{ds}\left(n\frac{d\mathbf{r}}{ds}\right) = \mathrm{grad}\, n, \tag{2}$$

where s is the length of the ray measured from a fixed point on it, and $\mathbf{r}$ is a position vector of a given point on the ray. For paraxial rays we have

$$n\frac{d^2x}{dz^2} = \frac{\partial}{\partial x}n = -n_2x,$$

and approximately

$$\frac{d^2x}{dz^2} + \frac{n_2}{n_0}x = 0. \tag{3}$$

The solution of Eq. (3) is

$$x = c_1\cos(n_2^{1/2}n_0^{-1/2}z) + c_2\sin(n_2^{1/2}n_0^{-1/2}z). \tag{4}$$

The slope of the ray at point z is

$$x' = \frac{dx}{dz} = -c_1(n_2^{1/2}n_0^{-1/2})\sin(n_2^{1/2}n_0^{-1/2}z) + c_2(n_2^{1/2}n_0^{-1/2})\cos(n_2^{1/2}n_0^{-1/2}z). \tag{5}$$

Denoting at $z = 0$ the distance between the ray and the axis by x_0 and the slope of the ray by x_0', we have

$$c_1 = x|_{z=0} = x_0,$$
$$c_2 = n_2^{-1/2}n_0^{1/2}x'|_{z=0} = n_2^{-1/2}n_0^{1/2}x_0'.$$

Substituting c_1 and c_2 into Eqs. (4) and (5), we obtain the *ABCD* matrix formula in a lenslike medium:

$$\begin{bmatrix} x \\ x' \end{bmatrix} = \begin{bmatrix} \cos(n_2^{1/2}n_0^{-1/2}z) & (n_2^{-1/2}n_0^{1/2}\sin(n_2^{1/2}n_0^{-1/2}z) \\ -(n_2^{1/2}n_0^{-1/2})\sin(n_2^{1/2}n_0^{-1/2}z) & \cos(n_2^{1/2}n_0^{-1/2}z) \end{bmatrix} \begin{bmatrix} x_0 \\ x_0' \end{bmatrix}. \tag{6}$$

To calculate the ray transfer matrix for a section of lenslike medium immersed in vacuum, we must invoke the help of Snell's law relevant to the slope of the rays at the section boundaries.[2] For paraxial rays we have approximately

$$\left.\begin{matrix} x_1' = n_0x_0' & x_2' = n_0x' \\ x_1 = x_0 & x_2 = x \end{matrix}\right\}, \tag{7}$$

where x_1' and x_2' are the slopes of the rays in vacuum at the input and output planes, respectively, and x_1 and x_2 are the distances of these rays from the optical axis. Substituting Eq. (7) into Eq. (6) one obtains the ray transfer matrix for a section of lenslike medium immersed in vacuum:

$$\begin{bmatrix} x_2 \\ x_2' \end{bmatrix} = \begin{bmatrix} \cos(n_2^{1/2}n_0^{-1/2}z) & (n_0n_2)^{-1/2}\sin(n_2^{1/2}n_0^{-1/2}z) \\ -(n_0n_2)^{1/2}\sin(n_2^{1/2}n_0^{-1/2}z) & \cos(n_2^{1/2}n_0^{-1/2}z) \end{bmatrix} \begin{bmatrix} x_1 \\ x_1' \end{bmatrix}, \tag{8}$$

where z is the length of the lenslike medium.

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Distributed Aperture and Useful Volume in the Laser Rod with Negative Lenses Ground on the Ends

It is assumed that the positive lensing effect of the thermally loaded laser rod is exactly compensated by the negative lenses. If the incident ray is parallel to the axis of the laser rod initially, it will also be parallel to the same axis when emerging from the other end of the rod. But the ray trace in the rod will be curved because of the divergence of the laser beam caused by the negative lens on the end of the laser rod and the convergence caused by the lenslike medium—the laser rod. At the same time, the effective aperture of the laser beam and the useful volume of the laser rod are decreased.[3] Assuming that the radius of curvature of two ends of the laser rod are equal to r, the ray transfer matrix in the input surface will be

$$\begin{bmatrix} x_0 \\ x_0' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \dfrac{n_0 - 1}{n_0 r} & \dfrac{1}{n_0} \end{bmatrix} \begin{bmatrix} x_1 \\ x_1' \end{bmatrix}. \tag{9}$$

Multiplying the matrix Eq. (6) by Eq. (9), one obtains

$$\begin{bmatrix} x \\ x' \end{bmatrix} = \begin{bmatrix} \cos(n_2^{1/2} n_0^{-1/2} z) + \dfrac{n_0 - 1}{n_0 r} (n_2^{-1/2} n_0^{1/2}) \sin(n_2^{1/2} n_0^{-1/2} z) & \dfrac{1}{n_0} (n_2^{-1/2} n_0^{1/2}) \sin(n_2^{1/2} n_0^{-1/2} z) \\ -(n_2^{1/2} n_0^{-1/2}) \sin(n_2^{1/2} n_0^{-1/2} z) + \dfrac{n_0 - 1}{n_0 r} \cos(n_2^{1/2} n_0^{-1/2} z) & \dfrac{1}{n_0} \cos(n_2^{1/2} n_0^{-1/2} z) \end{bmatrix} \begin{bmatrix} x_1 \\ x_1 \end{bmatrix}. \tag{10}$$

As shown by Barnes and Scalise,[3] the limiting ray osculates the lateral surface at the middle of the laser rod and is parallel to the axis. That is, when $z = l/2$ (l = the length of the laser rod), we have $x = a$ (a = the radius of the laser rod), and $x' = 0$. Since the incident ray is also parallel to the axis, we have $x_1' = 0$. Substituting these expressions into Eq. (10), we obtain

$$a = \Big\{ \cos[n_2^{1/2} n_0^{-1/2} (l/2)] + \frac{n_0 - 1}{n_0 r} (n_2^{-1/2} n_0^{1/2}) \sin[n_2^{1/2} n_0^{-1/2} (l/2)] \Big\} x_1, \tag{11}$$

$$O = \Big\{ -(n_2^{1/2} n_0^{-1/2}) \sin[n_2^{1/2} n_0^{-1/2} (l/2)] + \frac{n_0 - 1}{n_0 r} \cos[n_2^{1/2} n_0^{-1/2} (l/2)] \Big\} x_1. \tag{12}$$

From Eqs. (10)–(12) and according to the method used by Barnes and Scalise,[3] we obtained the limiting aperture of the ray and the useful volume of the laser rod, namely,

$$\frac{x_1}{a} = \left[1 + \frac{(n_0 - 1) l}{2 n_0 r}\right]^{-1/2}, \tag{13}$$

$$V \simeq \pi a^2 l \left[1 - \frac{(n_0 - 1) l}{6 n_0 r}\right]. \tag{14}$$

Equations (13) and (14) obtained by us differ from the result obtained in Ref. 3. In Ref. 3,

$$\frac{x_1}{a} = \left[1 + \frac{(n_0 - 1) l}{2 n_0^2 r}\right]^{-1/2},$$

$$V \simeq \pi a^2 l \left[1 - \frac{(n_0 - 1) l}{6 n_0^2 r}\right].$$

The reason is that the *ABCD* matrix (1) used in Ref. 3 is the ray transfer matrix for a section of lenslike medium immersed in vacuum [our Eq. (8)], and this is not correct. As mentioned above, when this matrix was introduced, one used Snell's law on the end of the rod, but in Ref. 3, when the distributed aperture and useful volume were introduced, Snell's law was used once again. We consider this repetition to be inappropriate. Changing the *ABCD* matrix (1) used in Ref. 3 in our Eq. (6), one can obtain the correct results.

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